Solving of $\frac{5\left(x^4+3\,x^2-1\right)}{6} = 0$

$\frac{5\left(x^4+3\,x^2-1\right)}{6} = 0$ is equivalent to the quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$ with:

$$\begin{array}{rcr} a & = & \frac{5}{6} \\ b & = & 0 \\ c & = & \frac{5}{2} \\ d & = & 0 \\ e & = & -\frac{5}{6} \end{array}$$

It is a quartic equation in quadratic form. We thus write $X = x^2$.

This takes us to the equation: $\frac{5\,X^2}{6} + \frac{5\,X}{2} - \frac{5}{6} = 0$. Its discriminant is equal to $\Delta = b^2 - 4ac = (\frac{5}{2})^2 - 4(\frac{5}{6})(-\frac{5}{6}) = \frac{325}{36} \simeq 9.02 > 0$.

This quadratic equation has thus two solutions:

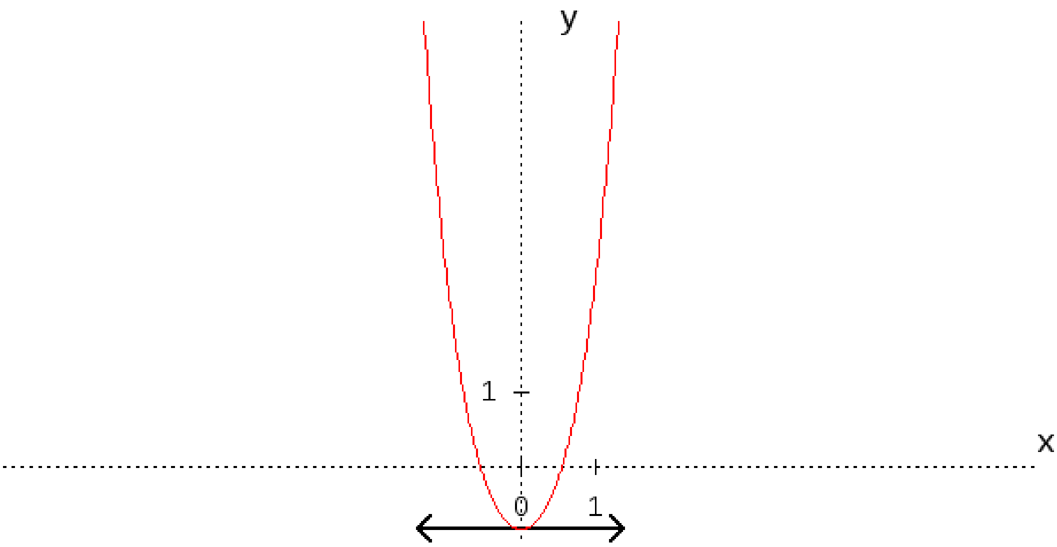
$$\begin{array}{rcccccc} X_1 & = & \frac{-b-\sqrt{\Delta}}{2a} & = & -\frac{\sqrt{13}}{2} - \frac{3}{2} & \simeq & -3.30 \\ X_2 & = & \frac{-b+\sqrt{\Delta}}{2a} & = & \frac{\sqrt{13}}{2} - \frac{3}{2} & \simeq & 0.302 \end{array}$$

Since $X_1 < 0$ and $X_2 > 0$, we get the real solutions:

$$\begin{array}{rcccccr} x_1 & = & \sqrt{X_2} & = & \frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}} & \simeq & 0.550 \\ x_2 & = & -\sqrt{X_2} & = & -\frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}} & \simeq & -0.550 \end{array}$$

All solutions in $\mathbb{R}$ are thus: $S = \{-\frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}}; \frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}}\}$.

Graphically, the curve having for equation $y = \frac{5\,x^4}{6} + \frac{5\,x^2}{2} - \frac{5}{6}$ is cutting the abscissa axis two times, at points having for abscissa x_1 and x_2.



Regarding complex solutions, we find in $\mathbb{C}$:

$$\begin{array}{rclclcr} z_1 & = & \sqrt{X_2} & = & \frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}} & \simeq & 0.550 \\ z_2 & = & -\sqrt{X_2} & = & -\frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}} & \simeq & -0.550 \\ z_3 & = & i\sqrt{-X_1} & = & \frac{\sqrt{\sqrt{13}+3}\,i}{\sqrt{2}} & \simeq & 1.81i \\ z_4 & = & -i\sqrt{-X_1} & = & -\frac{\sqrt{\sqrt{13}+3}\,i}{\sqrt{2}} & \simeq & -1.81i \end{array}$$

Thus $S = \{\frac{\sqrt{\sqrt{13}+3}\,i}{\sqrt{2}}; -\frac{\sqrt{\sqrt{13}+3}\,i}{\sqrt{2}}; \frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}}; -\frac{\sqrt{\sqrt{13}-3}}{\sqrt{2}}\}$.

<u>Note</u>: these results have been obtained from an automated program and are not guaranteed to be exact.

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